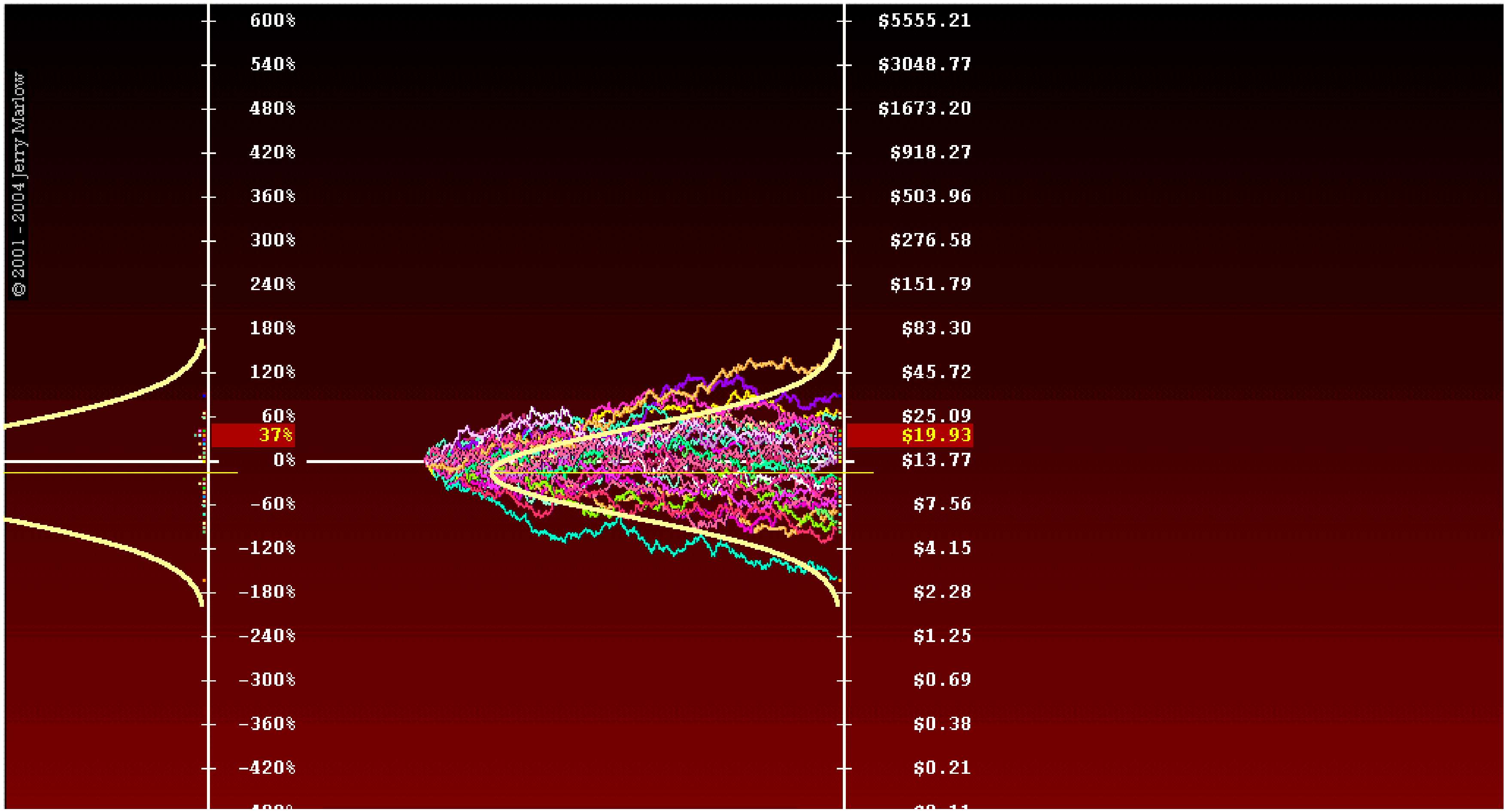
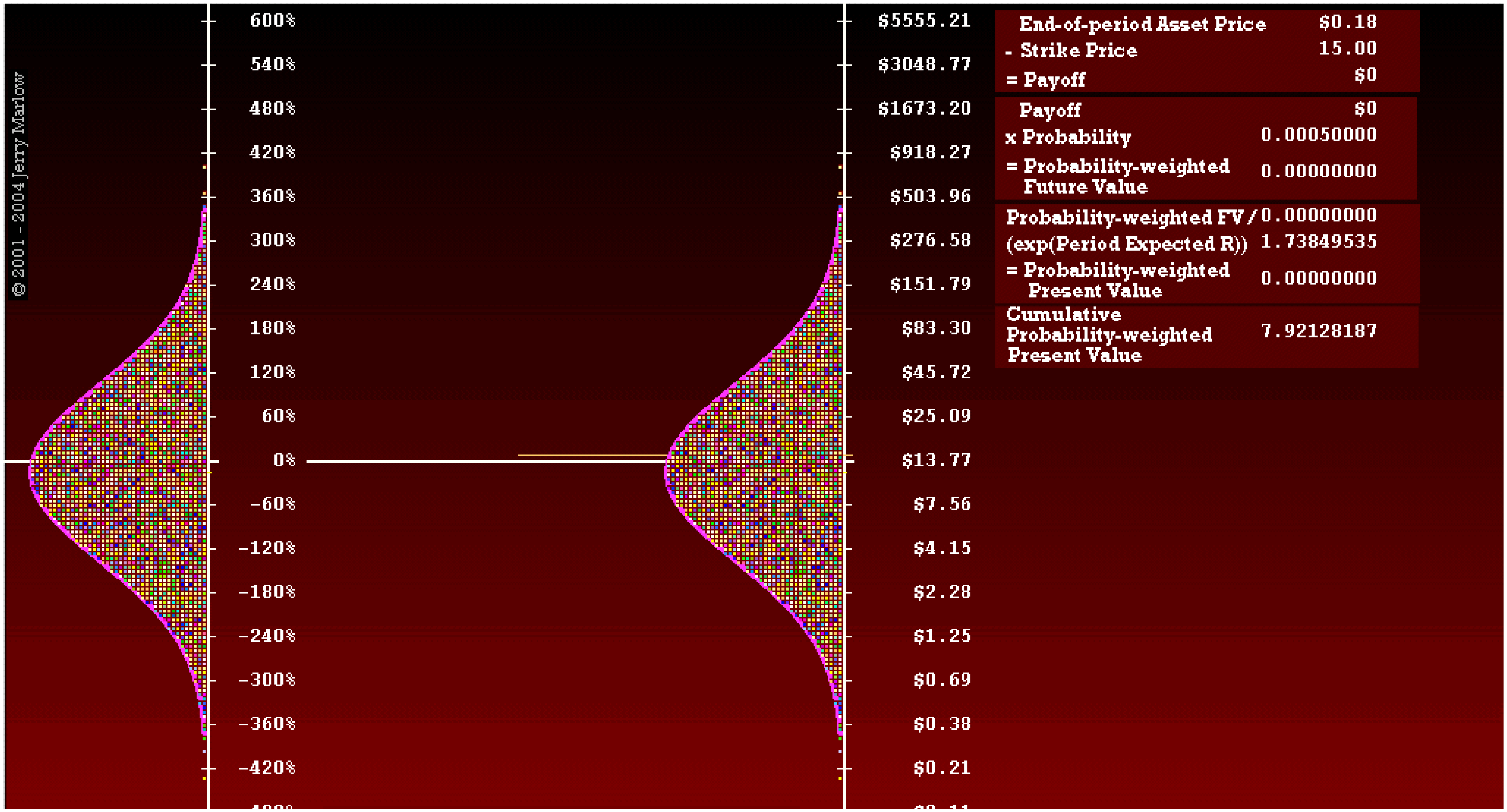


The spread of the bell-shaped curve and how high it sits on the price axis determines the option's value.

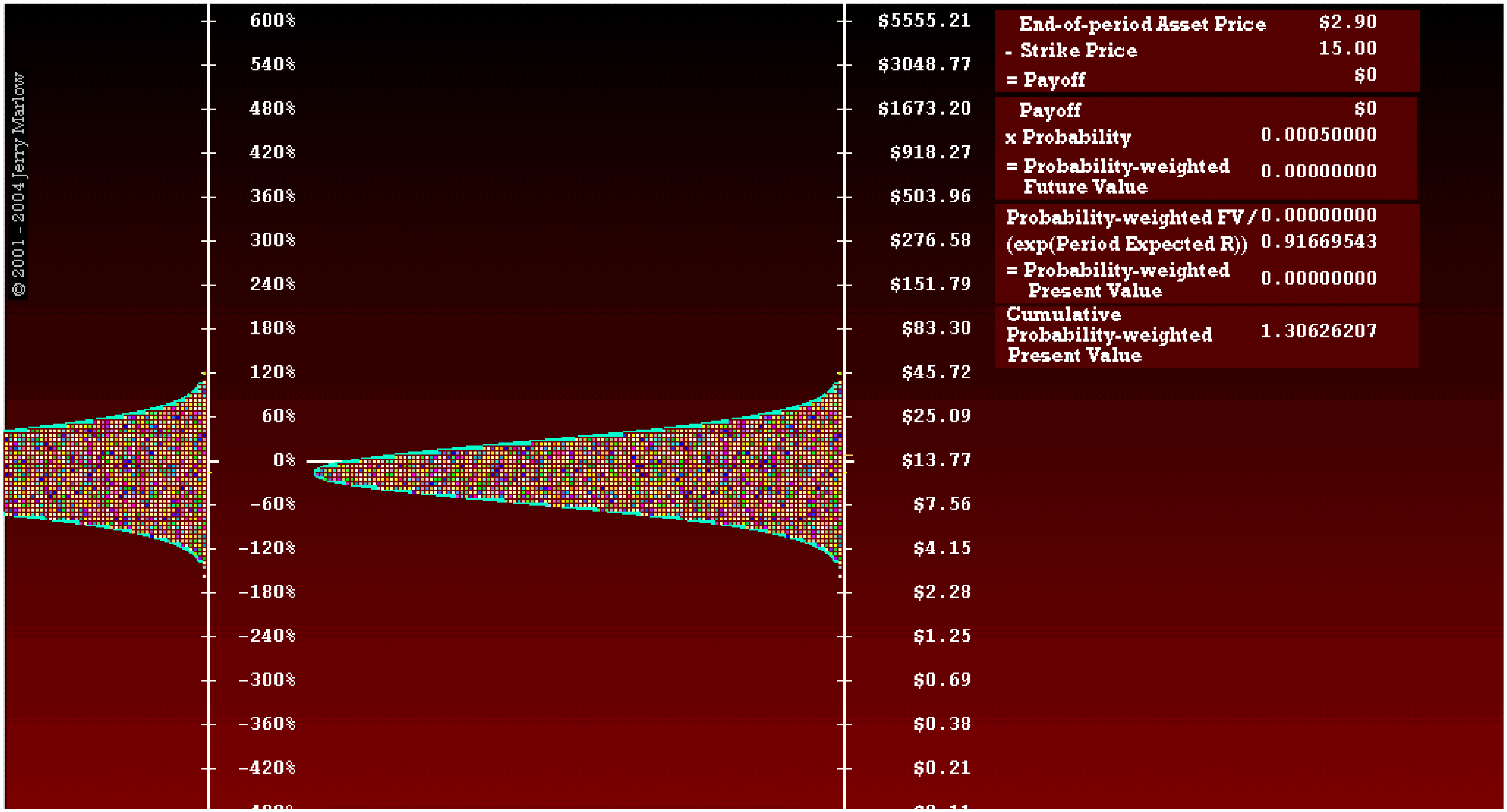
To value this option, we drew the stock forecast as a bell-shaped curve, filtered the curve through the option's strike price, and found the cumulative probability-weighted present value of the potential payoffs. Had the bell-shaped curve been higher, lower, more spread out or less spread out, the option would have had a different value. The spread of the bell-shaped curve and how high it sits on the price axis determines the option's value.



A more spread out bell-shaped curve centered at the same height gives a higher option value.

The stock forecast we evaluated previously gave an option value of $2.87. The bell-shaped curve shown here is centered at the same height on the stock-price axis but is more spread out.

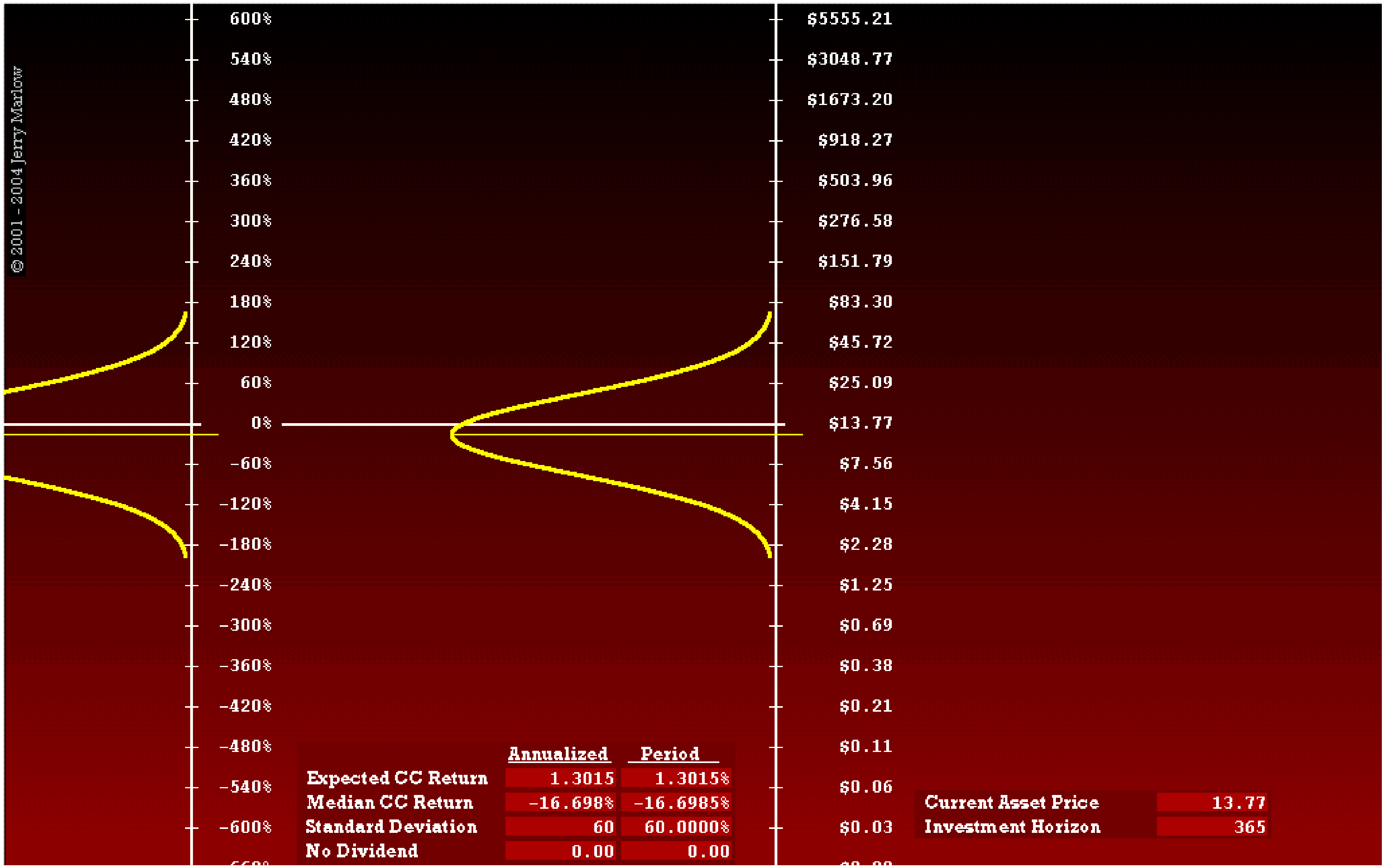
Accordingly more stock-price outcomes— more little squares— are above the strike price. They are farther above the strike price. There are more positive payoffs. The payoffs are larger. For the same $15.00 strike price, the cumulative probability-weighted present value of the potential payoffs is $7.92.



A less spread out bell-shaped curve centered at the same height gives a lower option value.

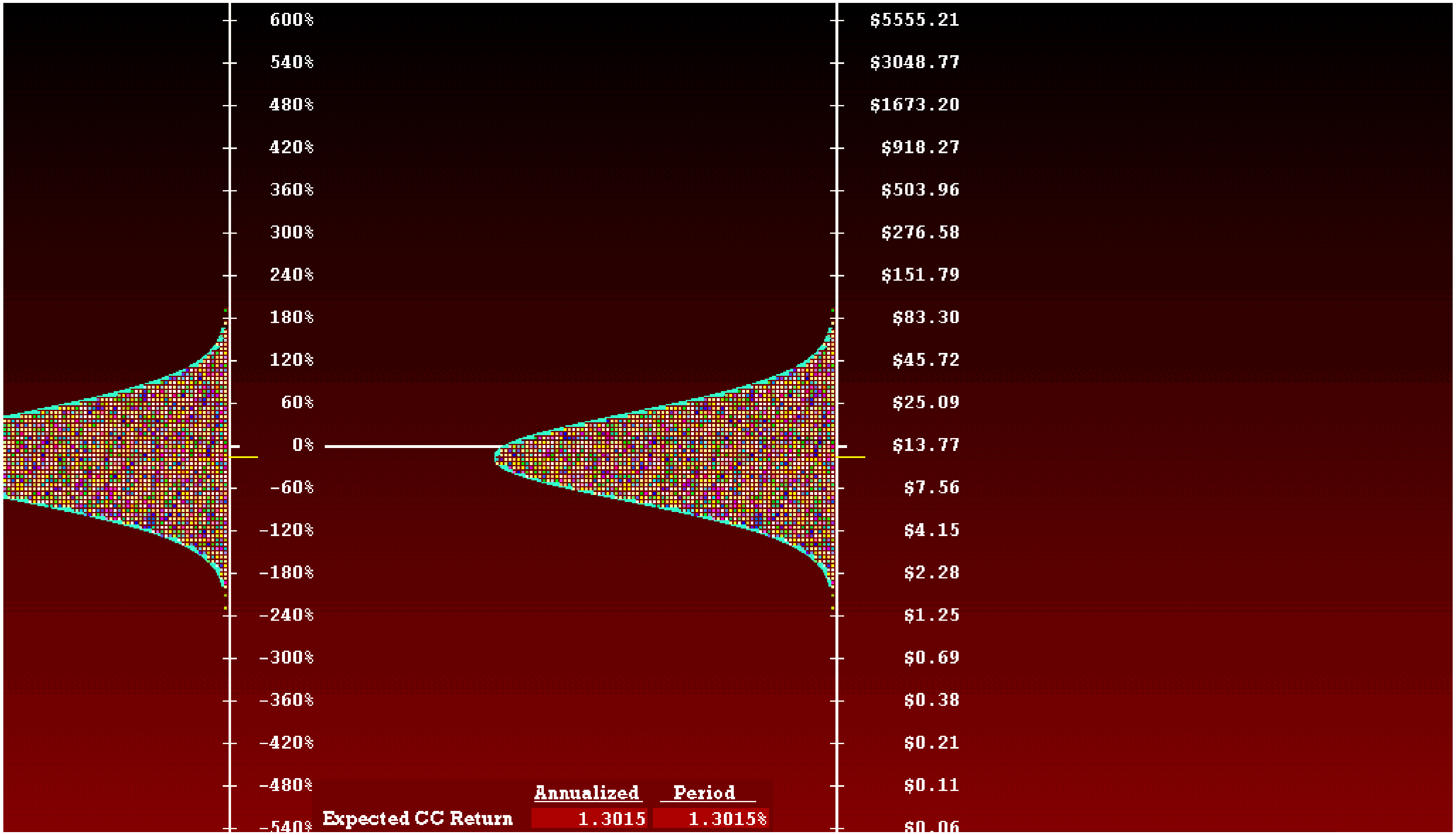
The bell-shaped curve shown here is centered at the same height on the stock-price axis but is less spread out. Fewer stock-price outcomes—fewer little squares— are above the strike price. Those that are above the strike price are not as far above.

For the same $15.00 strike price, the cumulative probability-weighted present value of the potential payoffs is $1.31.



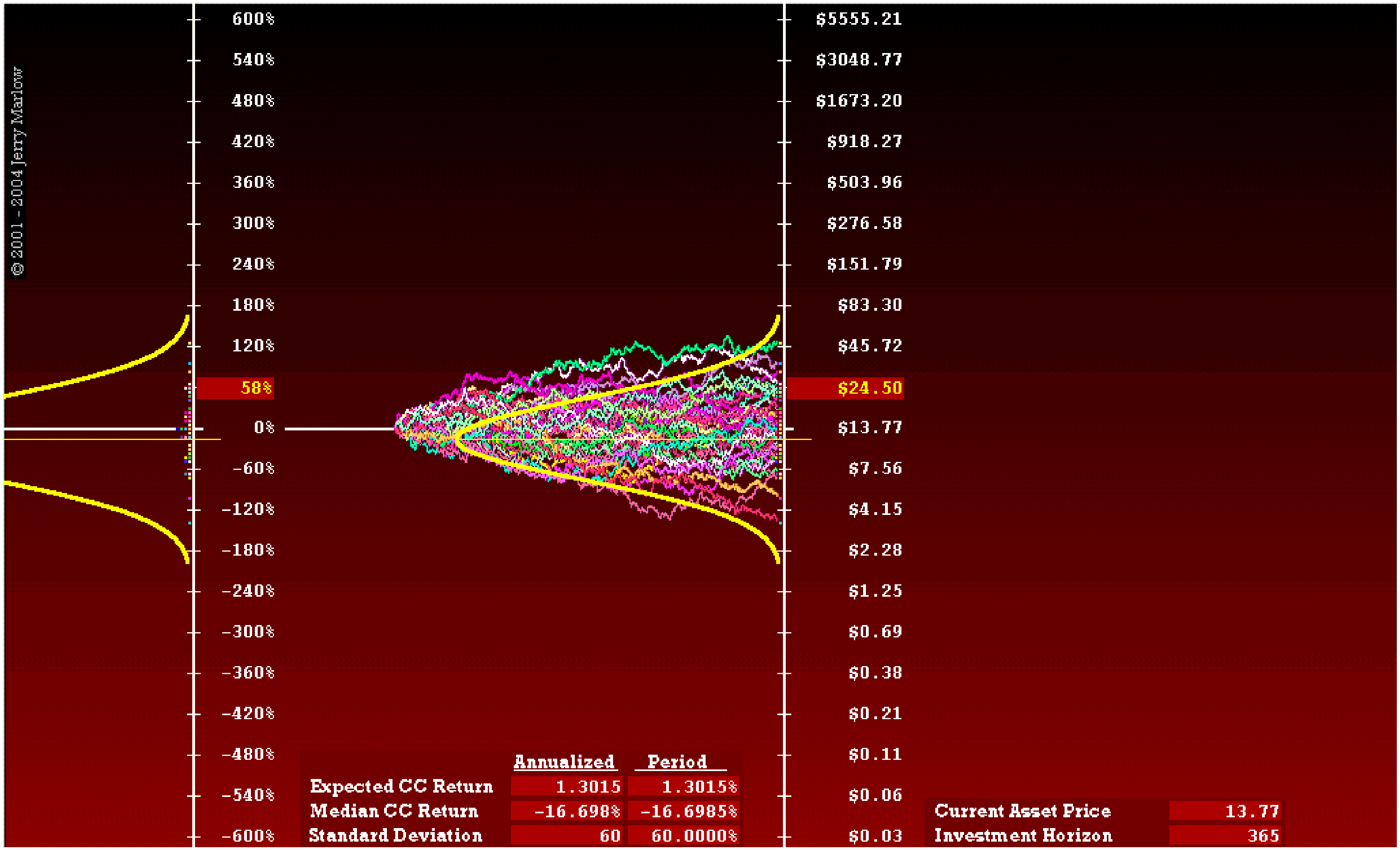
To draw a stock forecast, we need to know its expected return, standard deviation of expected volatility, current stock price, dividend forecast and the forecast's time horizon.

A stock's *return* forecast can be expressed in just two components: expected return and standard deviation of expected volatility. These are quoted in annual terms. To draw the forecast for a specific time horizon, we need to know its duration. To draw a price forecast, we also need to know the current stock price and the stock's dividend forecast.



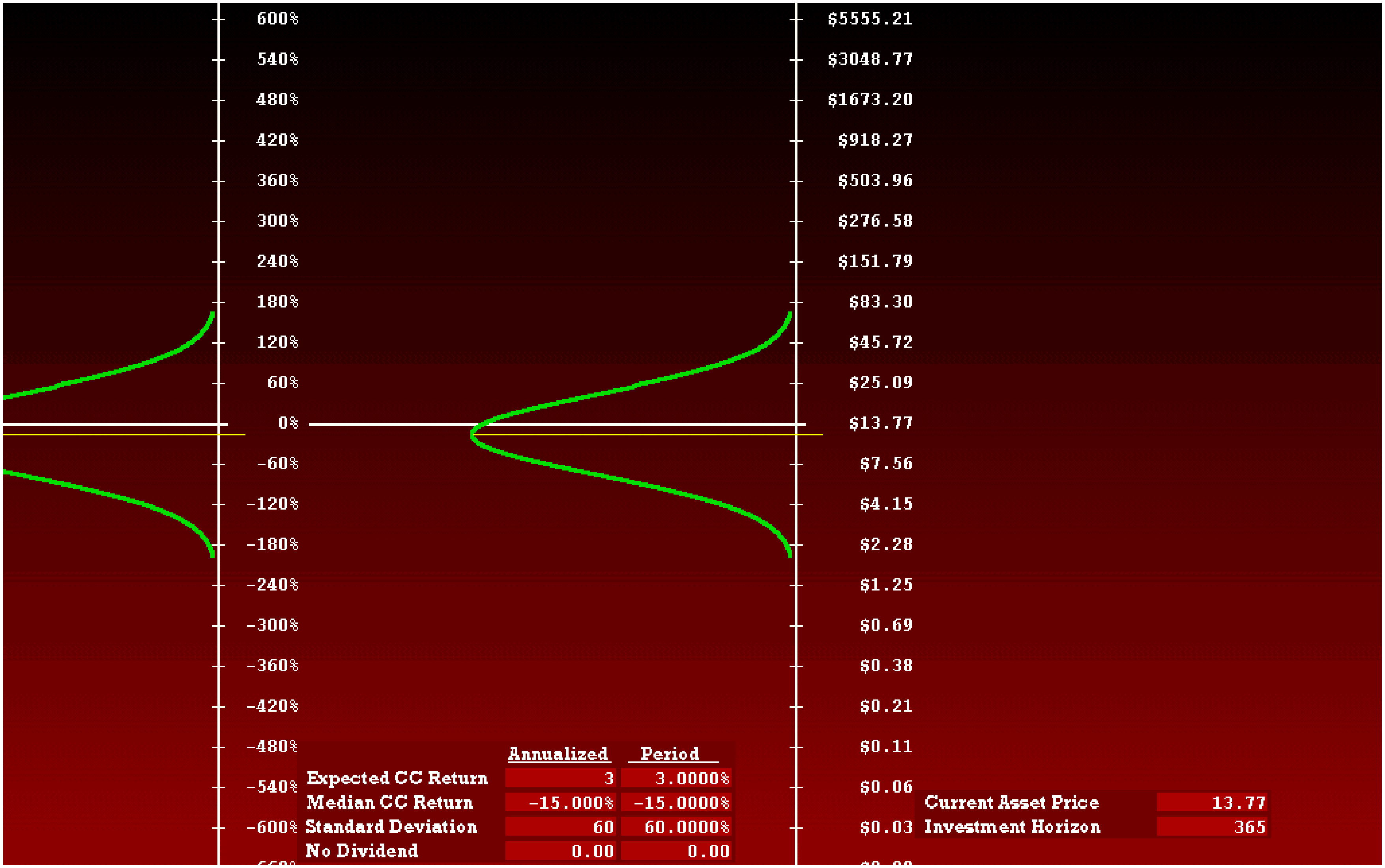
A stock's expected return is the average of all the returns in its probability distribution.

Expected return is the per-year *average* rate of return of all the little squares in the bell-shaped curve. In option valuation, we express expected return as a continuously compounded rate of return. In our initial valuation forecast, we used an expected return of 1.3015%, which happened to be equal to the then current one-year risk-free rate of return.



Standard deviation of expected volatility tells us how spread out the bell-shaped curve is.

The standard deviation of expected volatility quantifies how much we can expect the stock price to jump around. The greater the expected volatility, the greater the uncertainty about where the price will end up. The greater the uncertainty, the more spread out the bell-shaped probability distribution. In our initial valuation forecast, the standard deviation of expected volatility was 60%, as it is here.



In the bell-shaped curve, the median return is less than the average or expected return.

Here we draw a stock forecast with an expected return of 3% and a standard deviation of expected volatility of 60%. Yet, we see that the middle of the bell-shaped curve is down at -15%. More little squares are below than above the current stock price of $13.77.

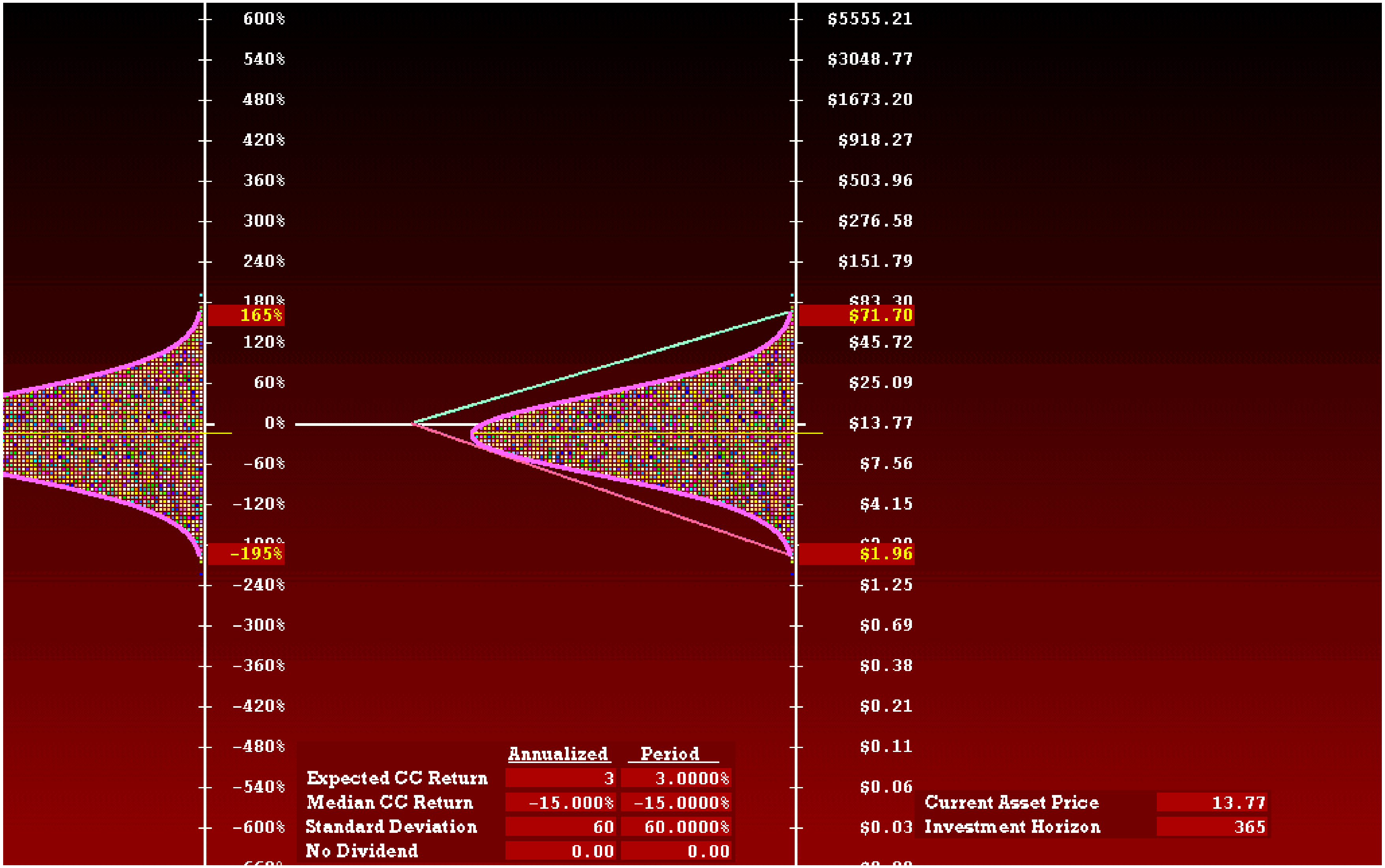
When working with continuously compounded rates of return, the average or expected return is not the same as the middle of the bell-shaped curve. To get a feel for why, imagine a portfolio of two stocks. The current price of each stock is \$100. The value of the portfolio is \$200. Over the course of a year, one stock has a continuously compounded return of 69%; the other has a continuously compounded return of –69%, that is, a loss of 69%.

The stock with a continuously compounded return of 69% doubles in value to \$200. The stock with a continuously compounded return of -69% loses half its value and goes to \$50. Hence, with one \$100 stock going up by 69% and the other going down by 69%, the value of the portfolio goes from \$200 to \$250— for an average continuously compounded return of 22.31%.

When working with bell-shaped curves and continuously compounded rates of return, expected return and the standard deviation of volatility combine to tell us where the middle of the bell-shaped curve will fall. The middle or median return is equal to the expected return minus half the standard deviation squared. In our example:

$$
\begin{aligned}
\text{Median Return} &= ER - .5(SD^2) \\
&= .03 - .5(.60^2) \\
&= .03 - .5(.36) \\
&= .03 - .18 \\
&= -.15 \\
&= -15\%
\end{aligned}
$$

If a forecast's median return is negative, as it is here, the forecast says that the probability that the stock price will go down is greater than the probability that it will go up. More little squares in the bell-shaped curve fall below the current price level than above it.



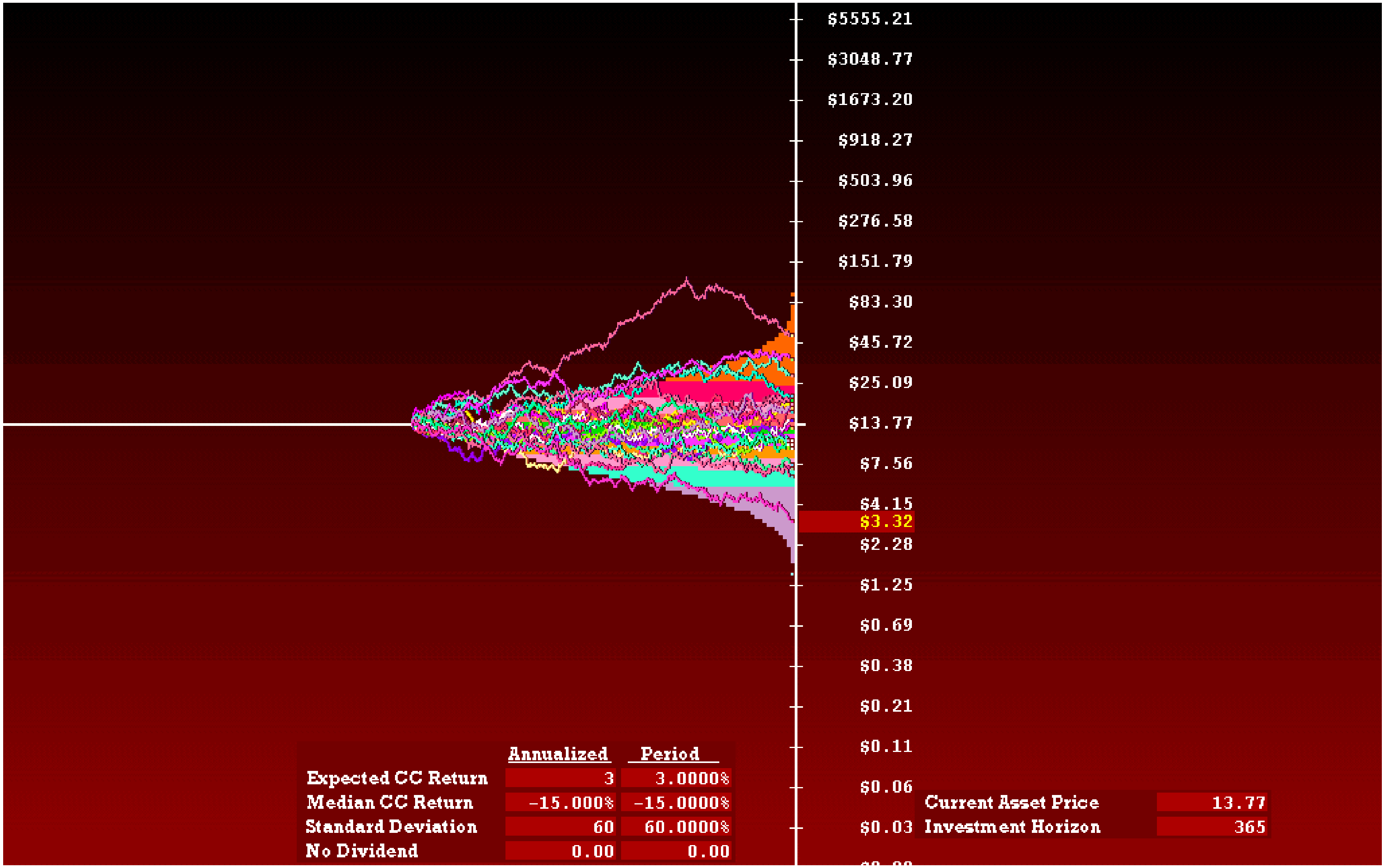
We can expect 99.87% of outcomes to fall between three standard deviations below the median return and three standard deviations above.

The forecast median return and standard deviation tell us the range within which we can be confident stock prices will be at the end of the forecast horizon. Under our statistical methodology, we can be 99.87% confident that the price will fall somewhere between three standard deviations below the median return and three standard deviations above.

In our example, we see that almost all the forecast returns fall above –195%, which is the median return minus three standard deviations:
-15% - (3 x 60%) = –195%.

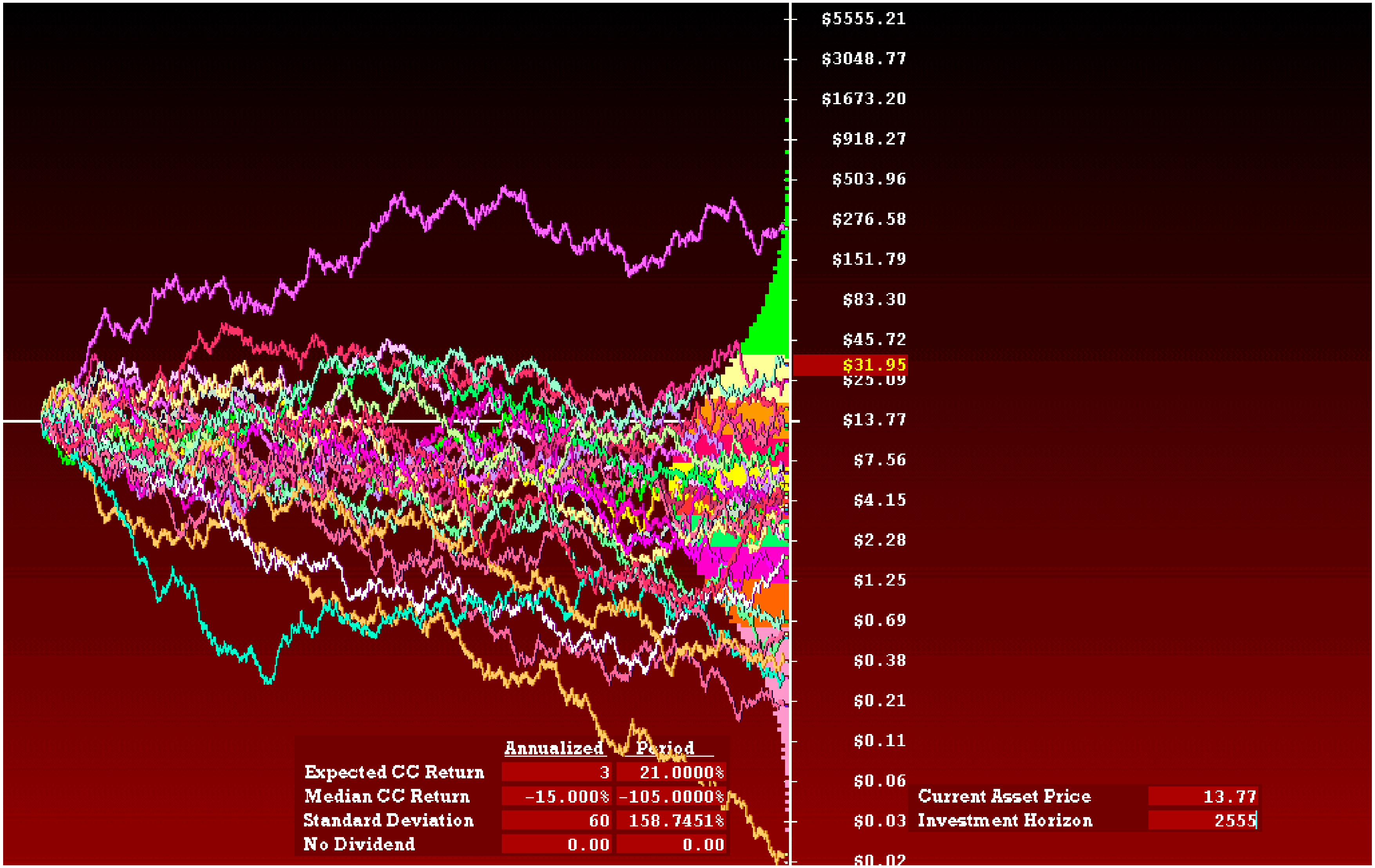
We see that almost all returns fall below 165%, which is the median return plus three standard deviations:
-15% + (3 x 60%) = 165%.

Looking across to the price axis, we see that the return range of -195% to 165% corresponds to a price range of $1.96 to $71.70. Hence, if we accept this forecast, we can be 99.87% confident that, one year from now, the price of this stock will be somewhere between $1.96 and $71.70.



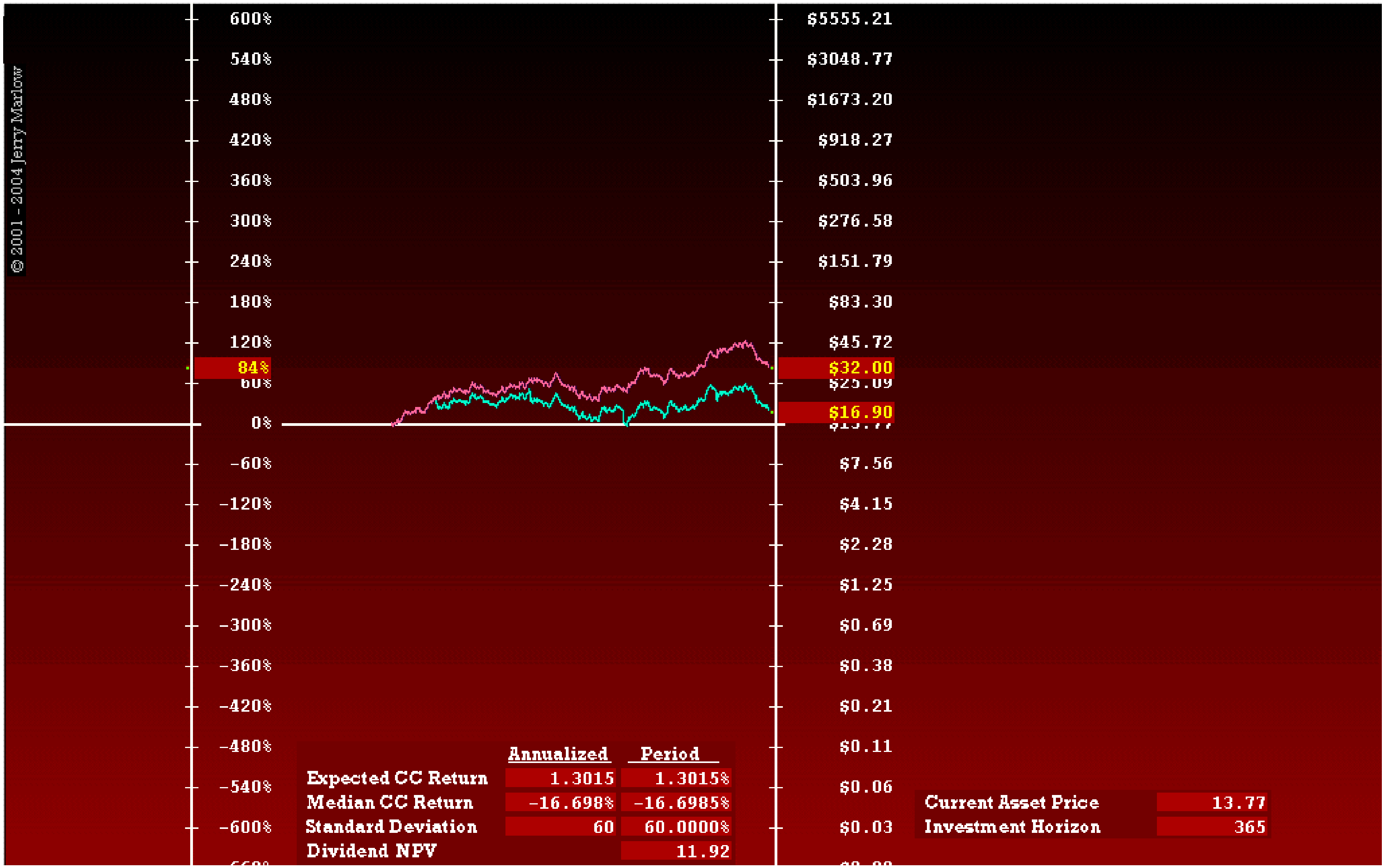
The longer the forecast horizon, the more spread out the bell-shaped curve.

As we do again here, we've been looking at price-path simulations and bell-shaped curves for one-year investment horizons. For a given standard deviation of expected volatility over a given investment horizon, a stock price has only so much time to jump around. It can go only so high or so low. The longer the investment horizon, the more time the stock price has to jump around, the higher or lower it can go, and the more spread out the bell-shaped curve.



The longer an option's time to expiration, the greater its value.

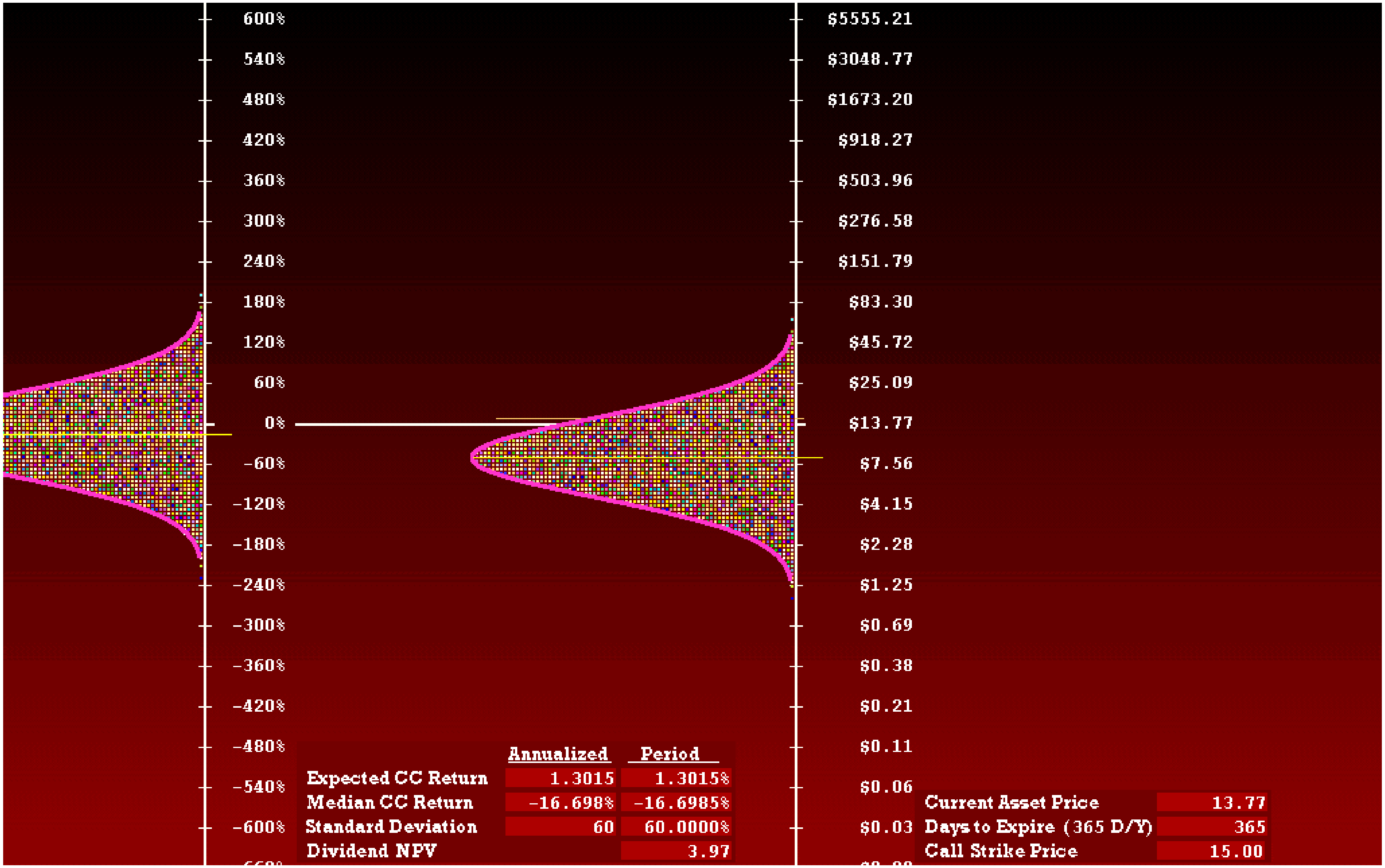
Here we look at the same forecast of expected return and volatility over a seven-year investment horizon. The stock price has more time to stray from where it is at time zero. In the bell-shaped curve, some of the potential price outcomes will be much farther above the strike price than they are in the same forecast over a one-year period. These more extreme potential payoffs give an option a value greater than that of one with a shorter time to expiration.



If a stock pays dividends, its price path will be lower than it otherwise would have been.

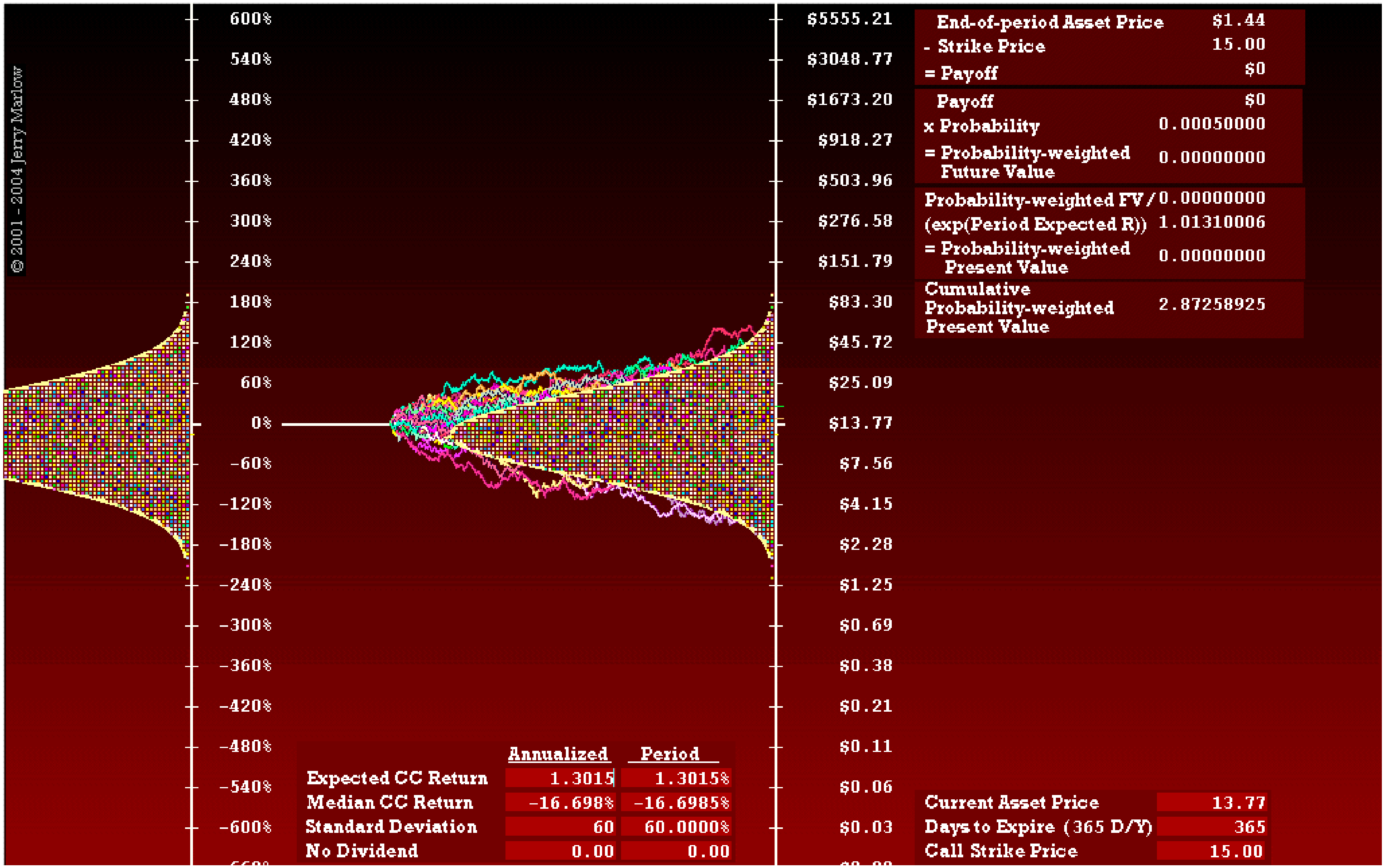
The forecast from which we simulate price paths and draw probability distributions is a return forecast. Stock returns include both price appreciation of the stock and dividend payments. Accordingly, any dividend payments come out of the price of the stock.

For sake of illustration, we simulate here a price path with and without four overly large quarterly dividend payments of $3.00 each.



If a stock pays dividends, the price bell-shaped curve will sit lower than the return curve.

Because dividend payments drop a stock's future price below what it otherwise would be, they shift the future-price bell-shaped curve below the return bell-shaped curve. All price outcomes in the forecast— all little squares— are lower than they otherwise would be. Option payoffs are based on stock price, not stock return. Hence, dividend payments lower call-option payoffs and values below what they otherwise would be.



This methodology allows us to value options fairly in the face of uncertainty.

Looking back on and rearranging what we have done, we see that, given a stock forecast of expected return and expected volatility and a discount rate, we can calculate an option's probability-weighted present value. That value takes into account all the prices that the stock *might* have at the end of the investment horizon. With this methodology, we can value options fairly in the face of uncertainty